Fully Private Grouped Matrix Multiplication with Colluding Workers

Lev Tauz and Lara Dolecek

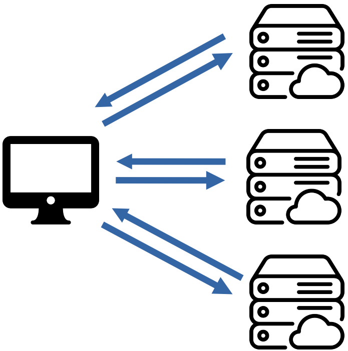
Electrical and Computer Engineering, University of California, Los Angeles, USA

15th Annual Non-Volatile Memories Workshop 2024

Outline

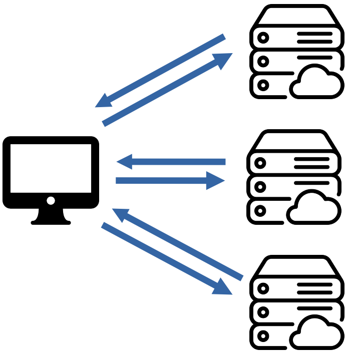
Modern Issues of Distributed Computing

- Distributed systems are a mainstay of modern big data applications due to high parallelization gains



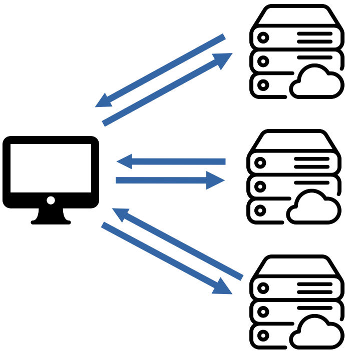
Modern Issues of Distributed Computing

- Distributed systems are a mainstay of modern big data applications due to high parallelization gains ... in theory



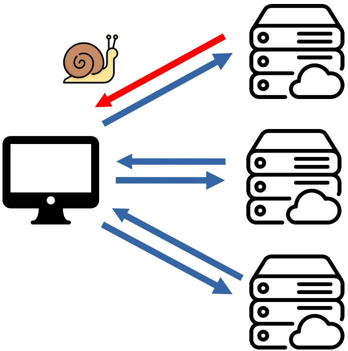
Modern Issues of Distributed Computing

- Distributed systems are a mainstay of modern big data applications due to high parallelization gains ... in theory
- With new pay-to-compute services, outsourcing work leads to new issues:



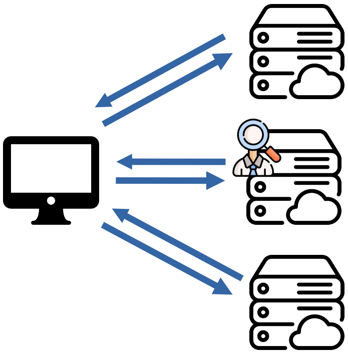
Modern Issues of Distributed Computing

- Distributed systems are a mainstay of modern big data applications due to high parallelization gains ... in theory
- With new pay-to-compute services, outsourcing work leads to new issues:
  - **Straggling Workers**



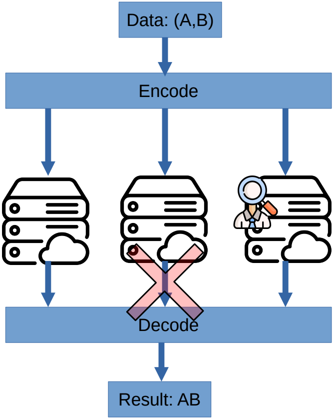
Modern Issues of Distributed Computing

- Distributed systems are a mainstay of modern big data applications due to high parallelization gains ... in theory
- With new pay-to-compute services, outsourcing work leads to new issues:
  - **Straggling Workers**
  - **Privacy Concerns**



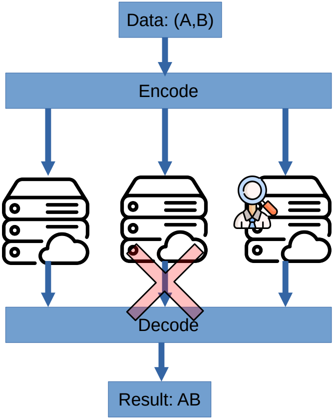
Coded Computation to the Rescue

- Coded computation addresses these issues by encoding data in a smart way



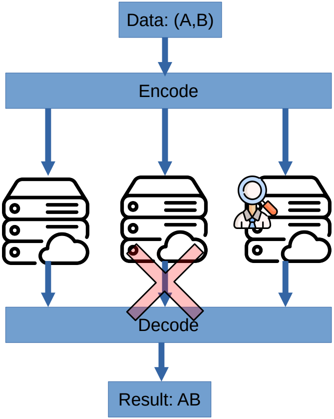
Coded Computation to the Rescue

- Coded computation addresses these issues by encoding data in a smart way
- Shown great success in **Distributed Large Matrix Multiplication**



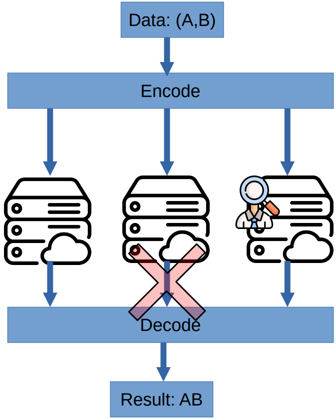
Coded Computation to the Rescue

- Coded computation addresses these issues by encoding data in a smart way
- Shown great success in **Distributed Large Matrix Multiplication**
  - Fundamental building block of modern machine learning algorithms



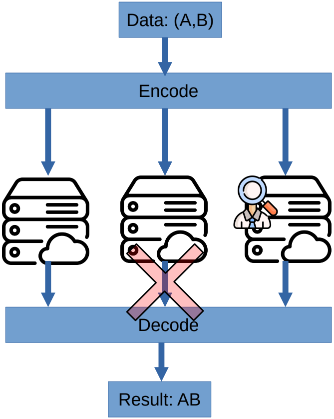
Coded Computation to the Rescue

- Coded computation addresses these issues by encoding data in a smart way
- Shown great success in **Distributed Large Matrix Multiplication**
  - Fundamental building block of modern machine learning algorithms
- In this work, we focus on private Distributed Large Matrix Multiplication as the primary use-case



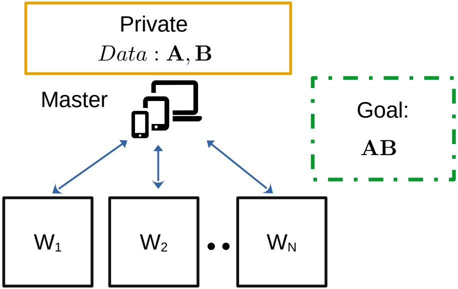
Coded Computation to the Rescue

- Coded computation addresses these issues by encoding data in a smart way
- Shown great success in **Distributed Large Matrix Multiplication**
  - Fundamental building block of modern machine learning algorithms
- In this work, we focus on private Distributed Large Matrix Multiplication as the primary use-case
- We start by discussing previously considered privacy models



Short Summary of Previous Privacy Models

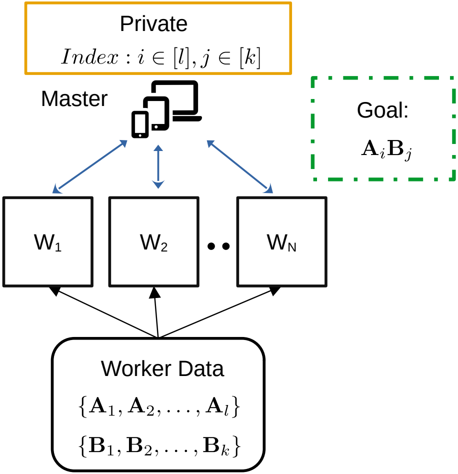
1. Data Privacy



Example:
Secure Matrix Multiplication
(Yu '20)

Short Summary of Previous Privacy Models

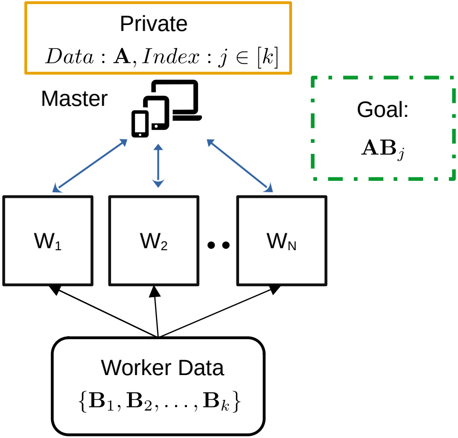
1. Data Privacy
2. Request Privacy



Example:
Fully Private Matrix Multiplication
(Kim '19)

Short Summary of Previous Privacy Models

1. Data Privacy
2. Request Privacy
3. Data + Request Privacy

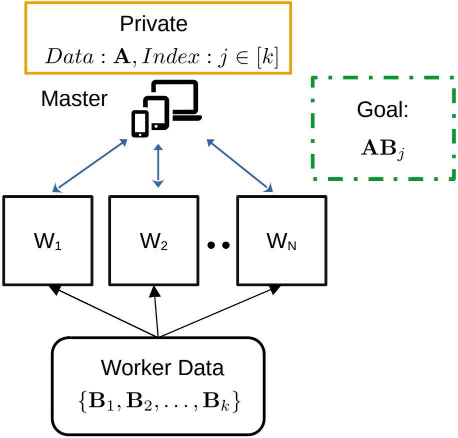


Example:
Secure and Private Matrix Multiplication
(Zhu '22)

Short Summary of Previous Privacy Models

1. Data Privacy
2. Request Privacy
3. Data + Request Privacy

New Privacy Model:
Batch Size Privacy



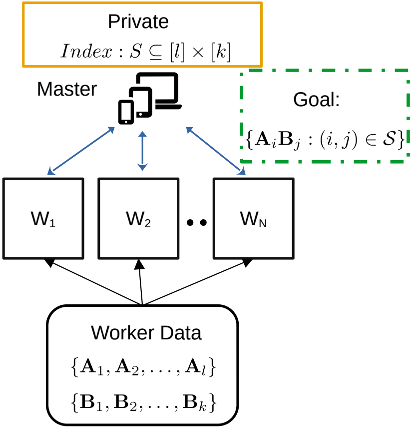
Example:
Secure and Private Matrix Multiplication
(Zhu '22)

New Privacy Model: Batch Size Privacy (BSP)

- The master wants to hide the **number of requests/batches** it wishes to compute

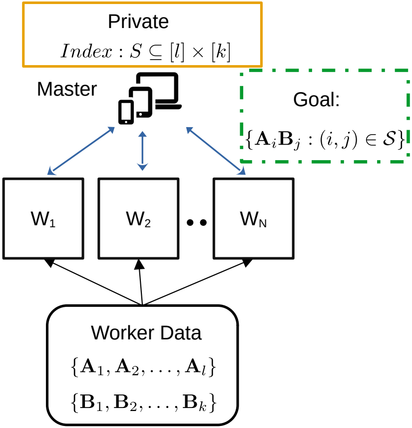
New Privacy Model: Batch Size Privacy (BSP)

- The master wants to hide the **number of requests/batches** it wishes to compute
- Example:



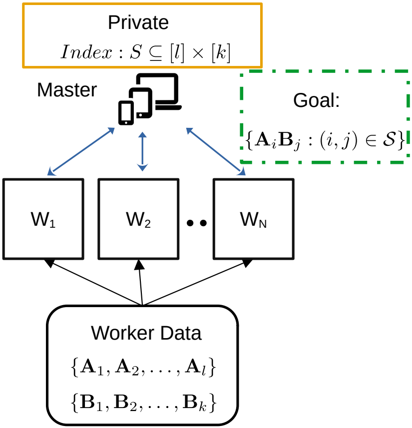
New Privacy Model: Batch Size Privacy (BSP)

- The master wants to hide the **number of requests/batches** it wishes to compute
- Example:
  - Consider a system where workers store two libraries $\{\mathbf{A}_1, \mathbf{A}_2\}$ and $\{\mathbf{B}_1\}$



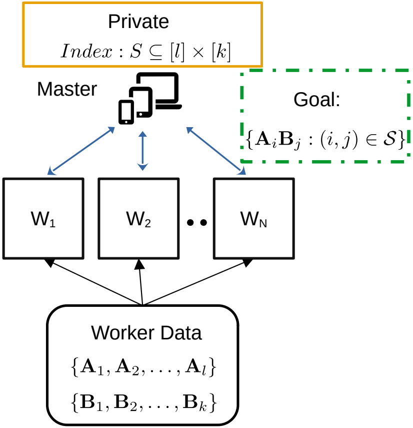
New Privacy Model: Batch Size Privacy (BSP)

- The master wants to hide the **number of requests/batches** it wishes to compute
- Example:
  - Consider a system where workers store two libraries $\{\mathbf{A}_1, \mathbf{A}_2\}$ and $\{\mathbf{B}_1\}$
  - The user can request any group of matrix products e.g. $\{\mathbf{A}_1\mathbf{B}_1\}$,$\{\mathbf{A}_2\mathbf{B}_1\}$, and $\{\mathbf{A}_1\mathbf{B}_1, \mathbf{A}_2\mathbf{B}_1\}$



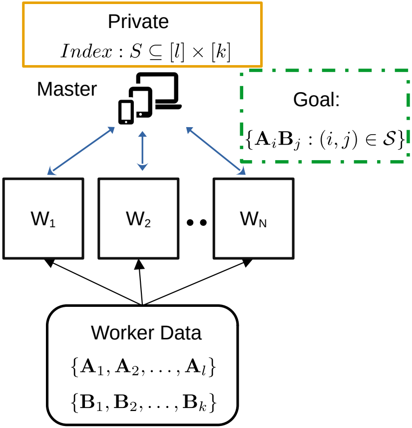
New Privacy Model: Batch Size Privacy (BSP)

- The master wants to hide the **number of requests/batches** it wishes to compute
- Example:
  - Consider a system where workers store two libraries $\{\mathbf{A}_1, \mathbf{A}_2\}$ and $\{\mathbf{B}_1\}$
  - The user can request any group of matrix products e.g. $\{\mathbf{A}_1\mathbf{B}_1\}$,$\{\mathbf{A}_2\mathbf{B}_1\}$, and $\{\mathbf{A}_1\mathbf{B}_1, \mathbf{A}_2\mathbf{B}_1\}$
  - If workers knew the batch size is 1, they reduce the space of possibilities



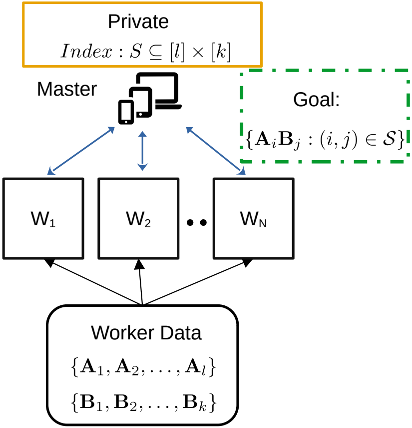
New Privacy Model: Batch Size Privacy (BSP)

- The master wants to hide the **number of requests/batches** it wishes to compute
- Example:
  - Consider a system where workers store two libraries $\{\mathbf{A}_1, \mathbf{A}_2\}$ and $\{\mathbf{B}_1\}$
  - The user can request any group of matrix products e.g. $\{\mathbf{A}_1\mathbf{B}_1\}$,$\{\mathbf{A}_2\mathbf{B}_1\}$, and $\{\mathbf{A}_1\mathbf{B}_1, \mathbf{A}_2\mathbf{B}_1\}$
  - If workers knew the batch size is 1, they reduce the space of possibilities
  - If workers knew the batch size is 2, they know the exact request



New Privacy Model: Batch Size Privacy (BSP)

- The master wants to hide the **number of requests/batches** it wishes to compute
- Example:
  - Consider a system where workers store two libraries $\{\mathbf{A}_1, \mathbf{A}_2\}$ and $\{\mathbf{B}_1\}$
  - The user can request any group of matrix products e.g. $\{\mathbf{A}_1\mathbf{B}_1\}$,$\{\mathbf{A}_2\mathbf{B}_1\}$, and $\{\mathbf{A}_1\mathbf{B}_1, \mathbf{A}_2\mathbf{B}_1\}$
  - If workers knew the batch size is 1, they reduce the space of possibilities
  - If workers knew the batch size is 2, they know the exact request

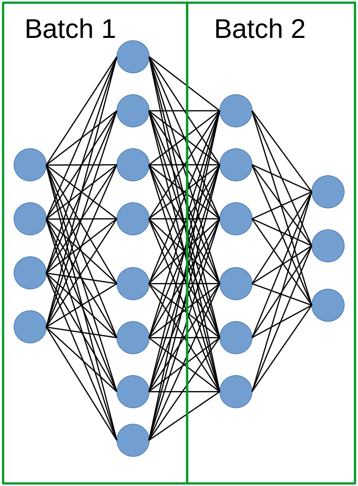


- Can even be *problematic* to systems without *batch size limits*

Practical Applications for Batch Size Privacy

Motivating Examples

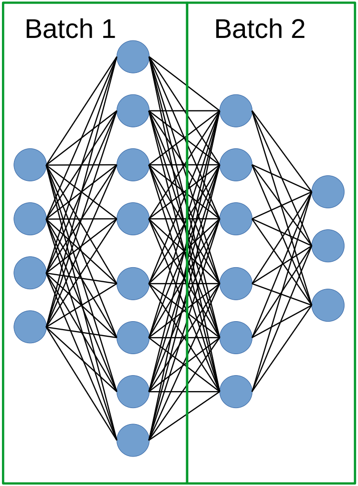
- **Inferring Size of Neural Networks**
  - Using batched coded computing, a user can train a network using model parallelism



Practical Applications for Batch Size Privacy

Motivating Examples

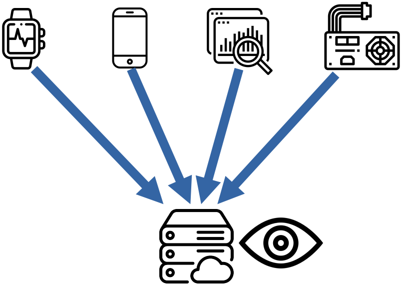
- **Inferring Size of Neural Networks**
  - Using batched coded computing, a user can train a network using model parallelism
  - Batch size can help infer the size of the network



Practical Applications for Batch Size Privacy

Motivating Examples

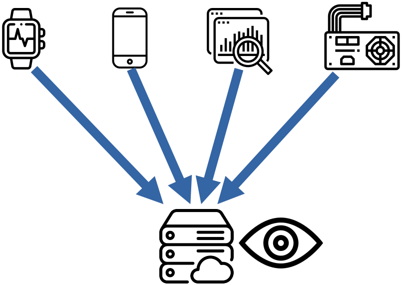
- **Identifying Entities in the System**



Practical Applications for Batch Size Privacy

Motivating Examples

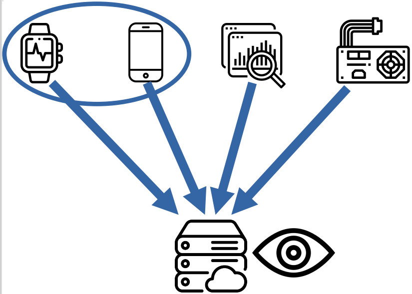
- **Identifying Entities in the System**
  - Batch size can help identify the identities of users or the use case of the request



Practical Applications for Batch Size Privacy

Motivating Examples

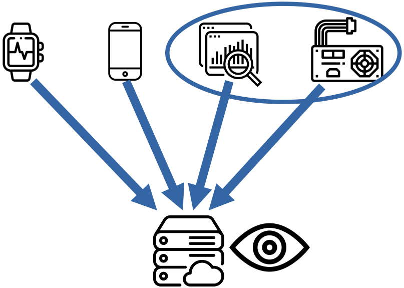
- **Identifying Entities in the System**
  - Batch size can help identify the identities of users or the use case of the request
  - Small IOT devices would generally request small batches



Practical Applications for Batch Size Privacy

Motivating Examples

- **Identifying Entities in the System**
  - Batch size can help identify the identities of users or the use case of the request
  - Small IOT devices would generally request small batches
  - Large analytic engines would request large batch sizes



Demonstrative Problem of Batch Size Privacy

- In this work, we consider a demonstrative problem setting that incorporates both **request and batch size privacy**

Demonstrative Problem of Batch Size Privacy

- In this work, we consider a demonstrative problem setting that incorporates both **request and batch size privacy**
- For simplicity, we consider a scenario where workers store all the data and, thus, no data privacy is necessary

Demonstrative Problem of Batch Size Privacy

- In this work, we consider a demonstrative problem setting that incorporates both **request and batch size privacy**
- For simplicity, we consider a scenario where workers store all the data and, thus, no data privacy is necessary
- The user is allowed to request any number of matrix products among the data stored at the workers

Demonstrative Problem of Batch Size Privacy

- In this work, we consider a demonstrative problem setting that incorporates both **request and batch size privacy**
- For simplicity, we consider a scenario where workers store all the data and, thus, no data privacy is necessary
- The user is allowed to request any number of matrix products among the data stored at the workers
- We name this new system **Fully Private Grouped Matrix Multiplication (FPGMM)**

Fully Private Grouped Matrix Multiplication (FPGMM)

- Assume workers store two libraries $\mathbf{A}_{[L_A]} = \{\mathbf{A}_i \in \mathbb{F}^{\alpha \times \alpha}, \forall i \in [L_A]\}$ and $\mathbf{B}_{[L_B]} = \{\mathbf{B}_i \in \mathbb{F}^{\alpha \times \alpha}, \forall i \in [L_B]\}$

Fully Private Grouped Matrix Multiplication (FPGMM)

- Assume workers store two libraries $\mathbf{A}_{[L_A]} = \{\mathbf{A}_i \in \mathbb{F}^{\alpha\times\alpha}, \forall i \in [L_A]\}$ and $\mathbf{B}_{[L_B]} = \{\mathbf{B}_i \in \mathbb{F}^{\alpha\times\alpha}, \forall i \in [L_B]\}$
- A master selects uniformly at random a non-empty set $\mathcal{S} \subseteq \{(i,j) : i \in [L_A], j \in [L_B]\}$

Fully Private Grouped Matrix Multiplication (FPGMM)

- Assume workers store two libraries $\mathbf{A}_{[L_A]} = \{\mathbf{A}_i \in \mathbb{F}^{\alpha \times \alpha}, \forall i \in [L_A]\}$ and $\mathbf{B}_{[L_B]} = \{\mathbf{B}_i \in \mathbb{F}^{\alpha \times \alpha}, \forall i \in [L_B]\}$
- A master selects uniformly at random a non-empty set $\mathcal{S} \subseteq \{(i,j) : i \in [L_A], j \in [L_B]\}$
- **Goal:** Calculate the matrix products $\mathbf{C}_{\mathcal{S}} \triangleq \{\mathbf{A}_i \mathbf{B}_j : (i,j) \in \mathcal{S}\}$ with the following requirements:

Fully Private Grouped Matrix Multiplication (FPGMM)

- Assume workers store two libraries $\mathbf{A}_{[L_A]} = \{\mathbf{A}_i \in \mathbb{F}^{\alpha\times\alpha}, \forall i \in [L_A]\}$ and $\mathbf{B}_{[L_B]} = \{\mathbf{B}_i \in \mathbb{F}^{\alpha\times\alpha}, \forall i \in [L_B]\}$
- A master selects uniformly at random a non-empty set $\mathcal{S} \subseteq \{(i,j) : i \in [L_A], j \in [L_B]\}$
- **Goal:** Calculate the matrix products $\mathbf{C}_{\mathcal{S}} \triangleq \{\mathbf{A}_i\mathbf{B}_j : (i,j) \in \mathcal{S}\}$ with the following requirements:

- **Small recovery threshold** R, i.e. the minimum number of worker results needed in order for the master to decode

Fully Private Grouped Matrix Multiplication (FPGMM)

- Assume workers store two libraries $\mathbf{A}_{[L_A]} = \{\mathbf{A}_i \in \mathbb{F}^{\alpha\times\alpha}, \forall i \in [L_A]\}$ and $\mathbf{B}_{[L_B]} = \{\mathbf{B}_i \in \mathbb{F}^{\alpha\times\alpha}, \forall i \in [L_B]\}$
- A master selects uniformly at random a non-empty set $\mathcal{S} \subseteq \{(i,j) : i \in [L_A], j \in [L_B]\}$
- **Goal:** Calculate the matrix products $\mathbf{C}_\mathcal{S} \triangleq \{\mathbf{A}_i\mathbf{B}_j : (i,j) \in \mathcal{S}\}$ with the following requirements:

- **Small recovery threshold** R, i.e. the minimum number of worker results needed in order for the master to decode
  - Measure of straggler resilience

Fully Private Grouped Matrix Multiplication (FPGMM)

- Assume workers store two libraries $\mathbf{A}_{[L_A]} = \{\mathbf{A}_i \in \mathbb{F}^{\alpha\times\alpha}, \forall i \in [L_A]\}$ and $\mathbf{B}_{[L_B]} = \{\mathbf{B}_i \in \mathbb{F}^{\alpha\times\alpha}, \forall i \in [L_B]\}$
- A master selects uniformly at random a non-empty set $\mathcal{S} \subseteq \{(i,j) : i \in [L_A], j \in [L_B]\}$
- **Goal:** Calculate the matrix products $\mathbf{C}_{\mathcal{S}} \triangleq \{\mathbf{A}_i\mathbf{B}_j : (i,j) \in \mathcal{S}\}$ with the following requirements:

- **Small recovery threshold** R, i.e. the minimum number of worker results needed in order for the master to decode
  - Measure of straggler resilience
- **Flexible computation and communication overhead**

Fully Private Grouped Matrix Multiplication (FPGMM)

- Assume workers store two libraries $\mathbf{A}_{[L_A]} = \{\mathbf{A}_i \in \mathbb{F}^{\alpha \times \alpha}, \forall i \in [L_A]\}$ and $\mathbf{B}_{[L_B]} = \{\mathbf{B}_i \in \mathbb{F}^{\alpha \times \alpha}, \forall i \in [L_B]\}$
- A master selects uniformly at random a non-empty set $\mathcal{S} \subseteq \{(i,j) : i \in [L_A], j \in [L_B]\}$
- **Goal:** Calculate the matrix products $\mathbf{C}_\mathcal{S} \triangleq \{\mathbf{A}_i \mathbf{B}_j : (i,j) \in \mathcal{S}\}$ with the following requirements:

- **Small recovery threshold** R, i.e. the minimum number of worker results needed in order for the master to decode
  - Measure of straggler resilience
- **Flexible computation and communication overhead**
- **Privacy against any** T **colluding workers**

Fully Private Grouped Matrix Multiplication (FPGMM)

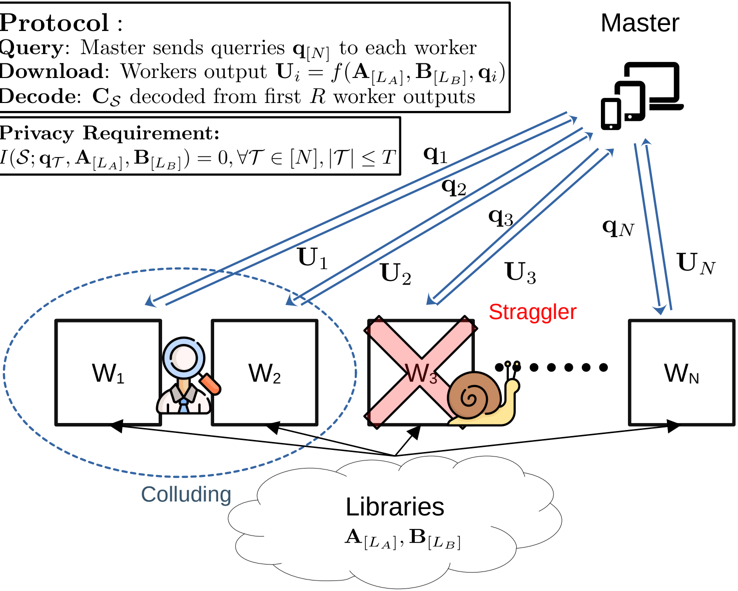
- Assume workers store two libraries $\mathbf{A}_{[L_A]} = \{\mathbf{A}_i \in \mathbb{F}^{\alpha\times\alpha}, \forall i \in [L_A]\}$ and $\mathbf{B}_{[L_B]} = \{\mathbf{B}_i \in \mathbb{F}^{\alpha\times\alpha}, \forall i \in [L_B]\}$
- A master selects uniformly at random a non-empty set $\mathcal{S} \subseteq \{(i,j) : i \in [L_A], j \in [L_B]\}$
- **Goal:** Calculate the matrix products $\mathbf{C}_{\mathcal{S}} \triangleq \{\mathbf{A}_i\mathbf{B}_j : (i,j) \in \mathcal{S}\}$ with the following requirements:

- **Small recovery threshold** R, i.e. the minimum number of worker results needed in order for the master to decode
  - Measure of straggler resilience
- **Flexible computation and communication overhead**
- **Privacy against any T colluding workers**
  - Any group of T workers *cannot learn anything about* $\mathcal{S}$ given their received queries, even the cardinality of $\mathcal{S}$

Model Overview

Protocol :
Query: Master sends querries $\mathbf{q}_{[N]}$ to each worker
Download: Workers output $\mathbf{U}_i = f(\mathbf{A}_{[L_A]}, \mathbf{B}_{[L_B]}, \mathbf{q}_i)$
Decode: $\mathbf{C}_{\mathcal{S}}$ decoded from first R worker outputs

Privacy Requirement:
$I(\mathcal{S}; \mathbf{q}_{\mathcal{T}}, \mathbf{A}_{[L_A]}, \mathbf{B}_{[L_B]}) = 0, \forall \mathcal{T} \in [N], |\mathcal{T}| \leq T$



Novelty of Problem

- In most coded computation settings, the workers are allowed to know something about the encoding process

Novelty of Problem

- In most coded computation settings, the workers are allowed to know something about the encoding process
  - For example, in polynomial coded based schemes, it is ok for workers to know the degree of the polynomial

Novelty of Problem

- In most coded computation settings, the workers are allowed to know something about the encoding process
  - For example, in polynomial coded based schemes, it is ok for workers to know the degree of the polynomial
- Since batch size impacts the encoding process, we need to **further limit the knowledge of the workers** to ensure BSP

Novelty of Problem

- In most coded computation settings, the workers are allowed to know something about the encoding process
  - For example, in polynomial coded based schemes, it is ok for workers to know the degree of the polynomial
- Since batch size impacts the encoding process, we need to **further limit the knowledge of the workers** to ensure BSP
- A related topic is *function privacy* which focused on:

Novelty of Problem

- In most coded computation settings, the workers are allowed to know something about the encoding process
  - For example, in polynomial coded based schemes, it is ok for workers to know the degree of the polynomial
- Since batch size impacts the encoding process, we need to **further limit the knowledge of the workers** to ensure BSP
- A related topic is *function privacy* which focused on:
  - Complex Polynomial Evaluations (Raviv '19)

Novelty of Problem

- In most coded computation settings, the workers are allowed to know something about the encoding process
  - For example, in polynomial coded based schemes, it is ok for workers to know the degree of the polynomial
- Since batch size impacts the encoding process, we need to **further limit the knowledge of the workers** to ensure BSP
- A related topic is *function privacy* which focused on:
  - Complex Polynomial Evaluations (Raviv '19)
  - Simple Linear Combinations (Sun '18)

Novelty of Problem

- In most coded computation settings, the workers are allowed to know something about the encoding process
  - For example, in polynomial coded based schemes, it is ok for workers to know the degree of the polynomial
- Since batch size impacts the encoding process, we need to **further limit the knowledge of the workers** to ensure BSP
- A related topic is *function privacy* which focused on:
  - Complex Polynomial Evaluations (Raviv '19)
  - Simple Linear Combinations (Sun '18)
- Our work falls between these two regimes since we focus on the bi-linear operation of matrix multiplication and batch processing

Novelty of Problem

- In most coded computation settings, the workers are allowed to know something about the encoding process
  - For example, in polynomial coded based schemes, it is ok for workers to know the degree of the polynomial
- Since batch size impacts the encoding process, we need to **further limit the knowledge of the workers** to ensure BSP
- A related topic is *function privacy* which focused on:
  - Complex Polynomial Evaluations (Raviv '19)
  - Simple Linear Combinations (Sun '18)
- Our work falls between these two regimes since we focus on the bi-linear operation of matrix multiplication and batch processing
- Additionally, in FPGMM, data can be re-used across multiple request, unlike other works that focus on distinct data

Novelty of Problem

- By further limiting the knowledge of the workers, many state-of-the-art coded computation schemes are *severly limited* in:

Novelty of Problem

- By further limiting the knowledge of the workers, many state-of-the-art coded computation schemes are *severly limited* in:
  - Optimizing the communication and computation costs

Novelty of Problem

- By further limiting the knowledge of the workers, many state-of-the-art coded computation schemes are *severly limited* in:
  - Optimizing the communication and computation costs (this paper)

Novelty of Problem

- By further limiting the knowledge of the workers, many state-of-the-art coded computation schemes are *severly limited* in:
  - Optimizing the communication and computation costs (this paper)
  - Providing other forms of privacy such as privacy from the master

Novelty of Problem

- By further limiting the knowledge of the workers, many state-of-the-art coded computation schemes are *severly limited* in:
  - Optimizing the communication and computation costs (this paper)
  - Providing other forms of privacy such as privacy from the master (ongoing work)

Outline

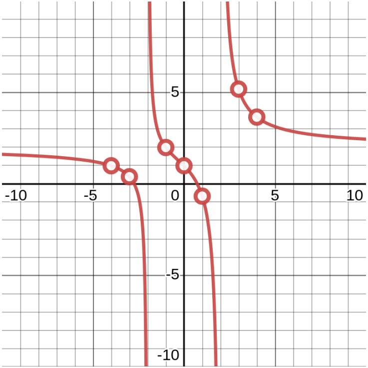
Useful Tool: Interpolation of Rational Functions

Rational Polynomial with Fixed Poles

$$F(z) = \sum_{i=1}^{N} \sum_{j=1}^{u_i} \frac{e_{i,j}}{(z - f_i)^j} + \sum_{j=0}^{K-M-1} e_{0,j} z^j$$

$f_1, f_2, \ldots, f_N$ are fixed

- Can interpolate $F(z)$ if the number of interpolation points is equal to the number of rational and polynomial terms (Gasca '89)



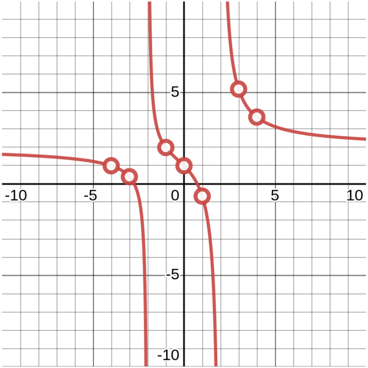
Useful Tool: Interpolation of Rational Functions

Rational Polynomial with Fixed Poles

$$F(z) = \sum_{i=1}^{N} \sum_{j=1}^{u_i} \frac{e_{i,j}}{(z - f_i)^j} + \sum_{j=0}^{K-M-1} e_{0,j} z^j$$

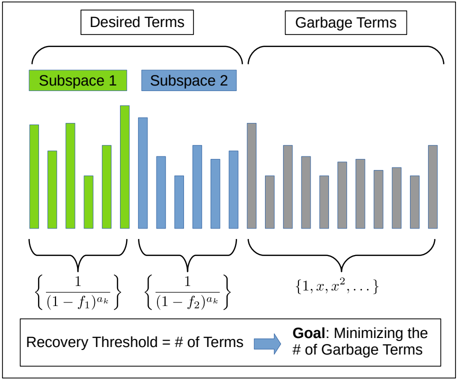
$f_1, f_2, \ldots, f_N$ are fixed

- Can interpolate $F(z)$ if the number of interpolation points is equal to the number of rational and polynomial terms (Gasca '89)
- Additionally, there are **fast** methods of interpolation that are comparable to polynomial interpolation (Olshevsky '01)



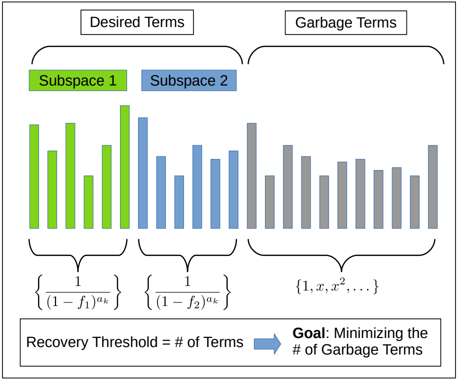
Another Tool: Cross-Subspace Alignment Codes

- Cross Subspace Alignment (CSA) codes are a coded computation scheme for calculating batches of matrix products



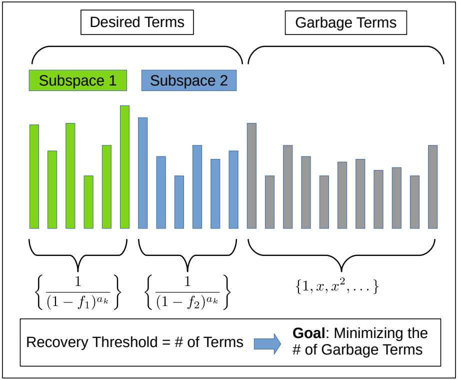
Another Tool: Cross-Subspace Alignment Codes

- Cross Subspace Alignment (CSA) codes are a coded computation scheme for calculating batches of matrix products
- Utilizes *rational functions* in order to encode the data and extract the desired terms

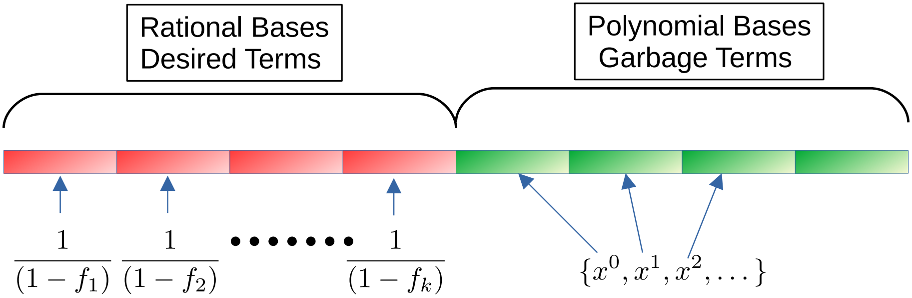


Another Tool: Cross-Subspace Alignment Codes

- Cross Subspace Alignment (CSA) codes are a coded computation scheme for calculating batches of matrix products
- Utilizes *rational functions* in order to encode the data and extract the desired terms
- Allows for **flexibility in communication and computation** costs due to its unique grouping capability

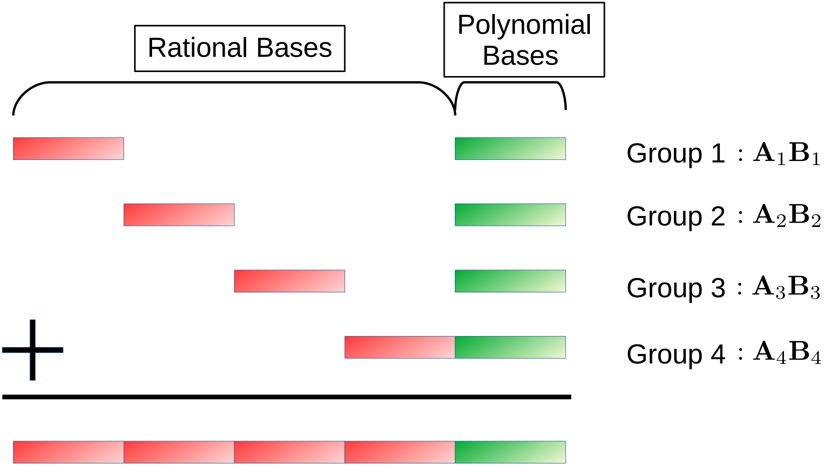


Grouping Structure of CSA codes

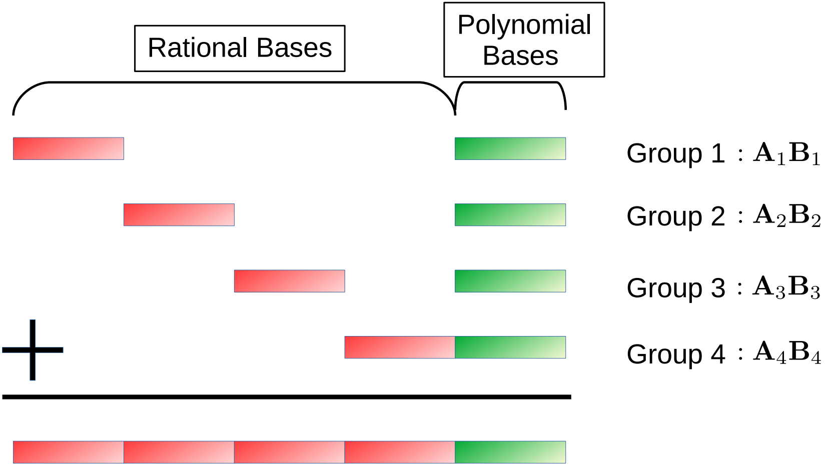


$$\mathcal{S} = \{\mathbf{A}_1\mathbf{B}_1, \mathbf{A}_2\mathbf{B}_2, \mathbf{A}_3\mathbf{B}_3, \mathbf{A}_4\mathbf{B}_4\}$$

Grouping Structure of CSA codes

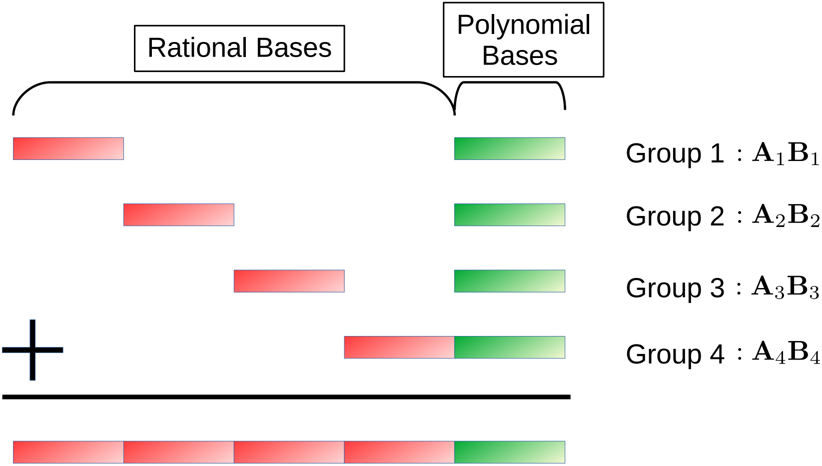


Grouping Structure of CSA codes



- Grouping reduces the recovery threshold at the cost of more computation

Grouping Structure of CSA codes



- Grouping reduces the recovery threshold at the cost of more computation
- **The number of groups reveals information about the batch size which breaks privacy**

Key Idea for Our Scheme

- Consider the following partitioning of the matrices given parameters m and n:

$$\mathbf{A}_i = \begin{bmatrix} \mathbf{A}_{i,1} \\ \vdots \\ \mathbf{A}_{i,m} \end{bmatrix} \forall i \in [L_A], \mathbf{B}_j = \begin{bmatrix} \mathbf{B}_{j,1} & \cdots & \mathbf{B}_{j,n} \end{bmatrix}, \forall j \in [L_B] \quad (1)$$

Key Idea for Our Scheme

- Consider the following partitioning of the matrices given parameters m and n:

$$\mathbf{A}_i = \begin{bmatrix} \mathbf{A}_{i,1} \\ \vdots \\ \mathbf{A}_{i,m} \end{bmatrix} \forall i \in [L_A], \mathbf{B}_j = \begin{bmatrix} \mathbf{B}_{j,1} & \cdots & \mathbf{B}_{j,n} \end{bmatrix}, \forall j \in [L_B] \qquad (1)$$

- If the master wants $\mathbf{A}_i\mathbf{B}_j$, then a sufficient condition is to get $\mathbf{A}_i\mathbf{B}_j = \{\mathbf{A}_{i,a}\mathbf{B}_{j,b}\}_{a\in[m],b\in[n]}$

Key Idea for Our Scheme

- Consider the following partitioning of the matrices given parameters m and n:

$$\mathbf{A}_i = \begin{bmatrix} \mathbf{A}_{i,1} \\ \vdots \\ \mathbf{A}_{i,m} \end{bmatrix} \forall i \in [L_A], \mathbf{B}_j = \begin{bmatrix} \mathbf{B}_{j,1} & \cdots & \mathbf{B}_{j,n} \end{bmatrix}, \forall j \in [L_B] \quad (1)$$

- If the master wants $\mathbf{A}_i\mathbf{B}_j$, then a sufficient condition is to get $\mathbf{A}_i\mathbf{B}_j = \{\mathbf{A}_{i,a}\mathbf{B}_{j,b}\}_{a\in[m],b\in[n]}$
- Hence, a sufficient condition to get $\mathbf{C}_{\mathcal{S}}$ is to calculate $\{\mathbf{A}_{i,q}\mathbf{B}_{j,s} : (i,j) \in \mathcal{S}, (q,s) \in [m] \times [n]\}$

Key Idea for Our Scheme

- Consider the following partitioning of the matrices given parameters m and n:

$$\mathbf{A}_i = \begin{bmatrix} \mathbf{A}_{i,1} \\ \vdots \\ \mathbf{A}_{i,m} \end{bmatrix} \forall i \in [L_A], \mathbf{B}_j = \begin{bmatrix} \mathbf{B}_{j,1} & \cdots & \mathbf{B}_{j,n} \end{bmatrix}, \forall j \in [L_B] \quad (1)$$

- If the master wants $\mathbf{A}_i\mathbf{B}_j$, then a sufficient condition is to get $\mathbf{A}_i\mathbf{B}_j = \{\mathbf{A}_{i,a}\mathbf{B}_{j,b}\}_{a\in[m],b\in[n]}$
- Hence, a sufficient condition to get $\mathbf{C}_{\mathcal{S}}$ is to calculate $\{\mathbf{A}_{i,q}\mathbf{B}_{j,s} : (i,j) \in \mathcal{S}, (q,s) \in [m] \times [n]\}$
- This is another instance of FPGMM but the batch size is now a multiple of mn

Grouping Independently of the Batch Size

- Let us re-index the submatrices as follows:

$$\widetilde{\mathbf{A}}_i = \mathbf{A}_{\lfloor \frac{i-1}{m} \rfloor + 1, (i-1 \mod m)+1}, i \in [mL_A]$$
$$\widetilde{\mathbf{B}}_j = \mathbf{B}_{\lfloor \frac{j-1}{n} \rfloor + 1, (j-1 \mod n)+1}, j \in [nL_B]$$

Grouping Independently of the Batch Size

- Let us re-index the submatrices as follows:

$$\widetilde{\mathbf{A}}_i = \mathbf{A}_{\lfloor \frac{i-1}{m} \rfloor + 1, (i-1 \mod m)+1}, i \in [mL_A]$$
$$\widetilde{\mathbf{B}}_j = \mathbf{B}_{\lfloor \frac{j-1}{n} \rfloor + 1, (j-1 \mod n)+1}, j \in [nL_B]$$

- We can define a new FPGMM problem with computation list $\widetilde{\mathcal{S}}$:

$$\widetilde{\mathcal{S}} = \{m(i-1)+q, n(j-1)+s : (i,j) \in \mathcal{S}, (q,s) \in [m] \times [n]\}$$

Grouping Independently of the Batch Size

- Let us re-index the submatrices as follows:

$$\widetilde{\mathbf{A}}_i = \mathbf{A}_{\lfloor \frac{i-1}{m} \rfloor + 1, (i-1 \mod m)+1}, i \in [mL_A]$$
$$\widetilde{\mathbf{B}}_j = \mathbf{B}_{\lfloor \frac{j-1}{n} \rfloor + 1, (j-1 \mod n)+1}, j \in [nL_B]$$

- We can define a new FPGMM problem with computation list $\widetilde{\mathcal{S}}$:

$$\widetilde{\mathcal{S}} = \{m(i-1)+q, n(j-1)+s : (i,j) \in \mathcal{S}, (q,s) \in [m] \times [n]\}$$

- **Fact:** $mn | \widetilde{\mathcal{S}}$

Grouping Independently of the Batch Size

- Let us re-index the submatrices as follows:

$$\widetilde{\mathbf{A}}_i = \mathbf{A}_{\lfloor \frac{i-1}{m} \rfloor + 1, (i-1 \mod m)+1}, i \in [mL_A]$$
$$\widetilde{\mathbf{B}}_j = \mathbf{B}_{\lfloor \frac{j-1}{n} \rfloor + 1, (j-1 \mod n)+1}, j \in [nL_B]$$

- We can define a new FPGMM problem with computation list $\widetilde{\mathcal{S}}$:

$$\widetilde{\mathcal{S}} = \{m(i-1)+q, n(j-1)+s : (i,j) \in \mathcal{S}, (q,s) \in [m] \times [n]\}$$

- **Fact:** $mn | \widetilde{\mathcal{S}}$
- **Key Idea:** We can group according to the partitioning parameters and not the original batch size

Grouping Independently of the Batch Size

- Let us re-index the submatrices as follows:

$$\widetilde{\mathbf{A}}_i = \mathbf{A}_{\lfloor \frac{i-1}{m} \rfloor + 1, (i-1 \mod m)+1}, i \in [mL_A]$$
$$\widetilde{\mathbf{B}}_j = \mathbf{B}_{\lfloor \frac{j-1}{n} \rfloor + 1, (j-1 \mod n)+1}, j \in [nL_B]$$

- We can define a new FPGMM problem with computation list $\widetilde{\mathcal{S}}$:

$$\widetilde{\mathcal{S}} = \{m(i-1)+q, n(j-1)+s : (i,j) \in \mathcal{S}, (q,s) \in [m] \times [n]\}$$

- **Fact:** $mn|\widetilde{\mathcal{S}}$
- **Key Idea:** We can group according to the partitioning parameters and not the original batch size
- Hence, we can still **achieve flexibility in overhead costs without compromising privacy**

Our Scheme

We breakdown our scheme into 3 main stages:

1. Encoding
2. Query and Computation
3. Decoding

Encoding Phase: Create Grouping

1. The master determines the *partitioning parameters* m and n and the *grouping parameter* r such that $r|mn$

Encoding Phase: Create Grouping

1. The master determines the *partitioning parameters* m and n and the *grouping parameter* r such that $r|mn$
2. The master groups $\widetilde{\mathcal{S}}$ into $\underline{r \text{ equal, non-overlapping groups}}$ of size $\delta = \frac{|\mathcal{S}|mn}{r}$ denoted by $\mathcal{Q}_1, \ldots, \mathcal{Q}_r$

Encoding Phase: Create Grouping

1. The master determines the *partitioning parameters* m and n and the *grouping parameter* r such that $r|mn$
2. The master groups $\widetilde{\mathcal{S}}$ into $\underline{r \text{ equal, non-overlapping groups}}$ of size $\delta = \frac{|\mathcal{S}|mn}{r}$ denoted by $\mathcal{Q}_1, \ldots, \mathcal{Q}_r$
3. For each $(i, j) \in \widetilde{\mathcal{S}}$, the master associates a distinct element $f_{i,j}$

Encoding Phase: Create Encoding Functions

4. The master then creates the encoding functions for $k \in [r]$:

$$a_{i,k}(x) = \omega_k(x)\left(\sum_{(q,s)\in\mathcal{Q}_k : q=i} \frac{1}{(x - f_{q,s})} + z^a_{i,k}(x)\right),$$

$$b_{j,k}(x) = \sum_{(q,s)\in\mathcal{Q}_k : s=j} \frac{1}{(x - f_{q,s})} + z^b_{j,k}(x)$$

$$\omega_k(x) = \prod_{(i,j)\in\mathcal{Q}_k} (x - f_{i,j})$$

Encoding Phase: Create Encoding Functions

4. The master then creates the encoding functions for $k \in [r]$:

$$a_{i,k}(x) = \omega_k(x) \left(\sum_{(q,s) \in \mathcal{Q}_k : q=i} \frac{1}{(x - f_{q,s})} + z^a_{i,k}(x) \right),$$

$$b_{j,k}(x) = \sum_{(q,s) \in \mathcal{Q}_k : s=j} \frac{1}{(x - f_{q,s})} + z^b_{j,k}(x)$$

$$\omega_k(x) = \prod_{(i,j) \in \mathcal{Q}_k} (x - f_{i,j})$$

- $z^a_{i,k}(x)$ and $z^b_{j,k}(x)$ are random polynomials of degree $T - 1$

Encoding Phase: Create Encoding Functions

4. The master then creates the encoding functions for $k \in [r]$:

$$a_{i,k}(x) = \omega_k(x) \left(\sum_{(q,s)\in\mathcal{Q}_k : q=i} \frac{1}{(x - f_{q,s})} + z^a_{i,k}(x) \right),$$

$$b_{j,k}(x) = \sum_{(q,s)\in\mathcal{Q}_k : s=j} \frac{1}{(x - f_{q,s})} + z^b_{j,k}(x)$$

$$\omega_k(x) = \prod_{(i,j)\in\mathcal{Q}_k} (x - f_{i,j})$$

- $z^a_{i,k}(x)$ and $z^b_{j,k}(x)$ are random polynomials of degree $T-1$
  - By **Shamir's secret sharing scheme**, ensures that any T evaluations of $a_{i,k}(x)$ and $b_{j,k}(x)$ are uniformly random variables

Encoding Phase: Create Encoding Functions

4. The master then creates the encoding functions for $k \in [r]$:

$$a_{i,k}(x) = \omega_k(x) \left(\sum_{(q,s)\in\mathcal{Q}_k : q=i} \frac{1}{(x - f_{q,s})} + z^a_{i,k}(x) \right),$$

$$b_{j,k}(x) = \sum_{(q,s)\in\mathcal{Q}_k : s=j} \frac{1}{(x - f_{q,s})} + z^b_{j,k}(x)$$

$$\omega_k(x) = \prod_{(i,j)\in\mathcal{Q}_k} (x - f_{i,j})$$

- $a_{i,k}(x)$ and $b_{j,k}(x)$ have the following property:

$$a_{i,k}(x)b_{j,k}(x) = \beta_{i,j,k}(x) + \begin{cases} \frac{\gamma^{i,j,k}}{(x-f_{i,j})} & (i,j) \in \mathcal{Q}_k, \\ 0 & \text{otherwise}, \end{cases}$$

where $\gamma^{i,j,k}$ is a non-zero constant and $\beta_{i,j,k}(x)$ is a polynomial of degree $\delta + 2T - 2$

Query and Computation

1. For worker $g \in [N]$, the master associates an element x_g that is distinct from $\{f_{i,j} : (i,j) \in \widetilde{\mathcal{S}}\}$

Query and Computation

1. For worker $g \in [N]$, the master associates an element x_g that is distinct from $\{f_{i,j} : (i, j) \in \widetilde{\mathcal{S}}\}$
2. The master then constructs a query for each worker that contains the following:

Query and Computation

1. For worker $g \in [N]$, the master associates an element x_g that is distinct from $\{f_{i,j} : (i,j) \in \widetilde{\mathcal{S}}\}$
2. The master then constructs a query for each worker that contains the following:

Query and Computation

1. For worker $g \in [N]$, the master associates an element x_g that is distinct from $\{f_{i,j} : (i,j) \in \widetilde{\mathcal{S}}\}$
2. The master then constructs a query for each worker that contains the following:

- The partitioning parameters m and n

Query and Computation

1. For worker $g \in [N]$, the master associates an element x_g that is distinct from $\{f_{i,j} : (i, j) \in \widetilde{\mathcal{S}}\}$
2. The master then constructs a query for each worker that contains the following:

- The partitioning parameters m and n
- The grouping parameter r

Query and Computation

1. For worker $g \in [N]$, the master associates an element x_g that is distinct from $\{f_{i,j} : (i,j) \in \widetilde{\mathcal{S}}\}$
2. The master then constructs a query for each worker that contains the following:

- The partitioning parameters m and n
- The grouping parameter r
- The encoding coefficients $\{\{a_{i,k}(x_g)\}_{i=1}^{mL_A}\}_{k=1}^{r}$, $\{\{b_{j,k}(x_g)\}_{j=1}^{nL_B}\}_{k=1}^{r}$

Query and Computation

1. For worker $g \in [N]$, the master associates an element x_g that is distinct from $\{f_{i,j} : (i,j) \in \widetilde{\mathcal{S}}\}$
2. The master then constructs a query for each worker that contains the following:

- The partitioning parameters m and n
- The grouping parameter r
- The encoding coefficients $\{\{a_{i,k}(x_g)\}_{i=1}^{mL_A}\}_{k=1}^{r}$, $\{\{b_{j,k}(x_g)\}_{j=1}^{nL_B}\}_{k=1}^{r}$

Any T collection of these queries does not reveal anything about $\mathcal{S}$

Query and Computation

3. The worker then partitions the matrices and encodes them using the encoding parameters:

$$\widehat{\mathbf{A}}_k = \sum_{i=1}^{mL_A} \widetilde{\mathbf{A}}_i a_{i,k}(x_g), \widehat{\mathbf{B}}_k = \sum_{j=1}^{nL_B} \widetilde{\mathbf{B}}_j b_{j,k}(x_g)$$

Query and Computation

3. The worker then partitions the matrices and encodes them using the encoding parameters:

$$\widehat{\mathbf{A}}_k = \sum_{i=1}^{mL_A} \widetilde{\mathbf{A}}_i a_{i,k}(x_g), \widehat{\mathbf{B}}_k = \sum_{j=1}^{nL_B} \widetilde{\mathbf{B}}_j b_{j,k}(x_g)$$

4. The worker computes and outputs:

$$\sum_{k=1}^{r} \widehat{\mathbf{A}}_k \widehat{\mathbf{B}}_k$$

Query and Computation

3. The worker then partitions the matrices and encodes them using the encoding parameters:

$$\widehat{\mathbf{A}}_k = \sum_{i=1}^{mL_A} \widetilde{\mathbf{A}}_i a_{i,k}(x_g), \widehat{\mathbf{B}}_k = \sum_{j=1}^{nL_B} \widetilde{\mathbf{B}}_j b_{j,k}(x_g)$$

4. The worker computes and outputs:

$$\sum_{k=1}^{r} \widehat{\mathbf{A}}_k \widehat{\mathbf{B}}_k = \sum_{k=1}^{r} \sum_{i=1}^{mL_A} \sum_{j=1}^{nL_B} \widetilde{\mathbf{A}}_i \widetilde{\mathbf{B}}_j a_{i,k}(x_g) b_{j,k}(x_g)$$

Query and Computation

3. The worker then partitions the matrices and encodes them using the encoding parameters:

$$\widehat{\mathbf{A}}_k = \sum_{i=1}^{mL_A} \widetilde{\mathbf{A}}_i a_{i,k}(x_g), \widehat{\mathbf{B}}_k = \sum_{j=1}^{nL_B} \widetilde{\mathbf{B}}_j b_{j,k}(x_g)$$

4. The worker computes and outputs:

$$\sum_{k=1}^{r} \widehat{\mathbf{A}}_k \widehat{\mathbf{B}}_k = \sum_{k=1}^{r} \sum_{i=1}^{mL_A} \sum_{j=1}^{nL_B} \widetilde{\mathbf{A}}_i \widetilde{\mathbf{B}}_j a_{i,k}(x_g) b_{j,k}(x_g)$$

$$= \sum_{k=1}^{r} \left(\sum_{(i,j)\in\mathcal{Q}_k} \frac{\gamma^{i,j,k} \widetilde{\mathbf{A}}_i \widetilde{\mathbf{B}}_j}{(x_g - f_{i,j})} + \sum_{i=1}^{mL_A} \sum_{j=1}^{nL_B} \widetilde{\mathbf{A}}_i \widetilde{\mathbf{B}}_j \beta_{i,j,k}(x_g) \right)$$

$$= \sum_{(i,j)\in\widetilde{\mathcal{S}}} \frac{\gamma^{i,j,k_{i,j}} \widetilde{\mathbf{A}}_i \widetilde{\mathbf{B}}_j}{(x_g - f_{i,j})} + \mathbf{I}(x_g)$$

where $\mathbf{I}(x)$ is a polynomial matrix of maximum degree $\delta + 2T - 2$

Query and Computation

3. The worker then partitions the matrices and encodes them using the encoding parameters:

$$\widehat{\mathbf{A}}_k = \sum_{i=1}^{mL_A} \widetilde{\mathbf{A}}_i a_{i,k}(x_g), \widehat{\mathbf{B}}_k = \sum_{j=1}^{nL_B} \widetilde{\mathbf{B}}_j b_{j,k}(x_g)$$

4. The worker computes and outputs:

$$\sum_{k=1}^{r} \widehat{\mathbf{A}}_k \widehat{\mathbf{B}}_k = \sum_{k=1}^{r} \sum_{i=1}^{mL_A} \sum_{j=1}^{nL_B} \widetilde{\mathbf{A}}_i \widetilde{\mathbf{B}}_j a_{i,k}(x_g) b_{j,k}(x_g)$$

$$= \sum_{k=1}^{r} \left(\sum_{(i,j)\in\mathcal{Q}_k} \frac{\gamma^{i,j,k} \widetilde{\mathbf{A}}_i \widetilde{\mathbf{B}}_j}{(x_g - f_{i,j})} + \sum_{i=1}^{mL_A} \sum_{j=1}^{nL_B} \widetilde{\mathbf{A}}_i \widetilde{\mathbf{B}}_j \beta_{i,j,k}(x_g) \right)$$

$$= \sum_{(i,j)\in\widetilde{\mathcal{S}}} \frac{\gamma^{i,j,k_{i,j}} \widetilde{\mathbf{A}}_i \widetilde{\mathbf{B}}_j}{(x_g - f_{i,j})} + \mathbf{I}(x_g)$$

where $\mathbf{I}(x)$ is a polynomial matrix of maximum degree $\delta + 2T - 2$

Decoding utilizing Rational Function Interpolation

1. The master now gets evaluations of the following function

$$\sum_{(i,j)\in\widetilde{\mathcal{S}}} \frac{\gamma^{i,j,k}\widetilde{\mathbf{A}}_i\widetilde{\mathbf{B}}_j}{(x-f_{i,j})} + \mathbf{I}(x)$$

Decoding utilizing Rational Function Interpolation

1. The master now gets evaluations of the following function

$$\sum_{(i,j)\in\widetilde{\mathcal{S}}} \frac{\gamma^{i,j,k}\widetilde{\mathbf{A}}_i\widetilde{\mathbf{B}}_j}{(x-f_{i,j})} + \mathbf{I}(x)$$

2. This function has $|\widetilde{\mathcal{S}}| = |\mathcal{S}|mn$ rational terms and $\delta + 2T - 1 = \frac{|\mathcal{S}|mn}{r} + 2T - 1$ polynomial terms

Decoding utilizing Rational Function Interpolation

1. The master now gets evaluations of the following function

$$\sum_{(i,j)\in\widetilde{\mathcal{S}}} \frac{\gamma^{i,j,k}\widetilde{\mathbf{A}}_i\widetilde{\mathbf{B}}_j}{(x - f_{i,j})} + \mathbf{I}(x)$$

2. This function has $|\widetilde{\mathcal{S}}| = |\mathcal{S}|mn$ rational terms and $\delta + 2T - 1 = \frac{|\mathcal{S}|mn}{r} + 2T - 1$ polynomial terms
3. Can be interpolated from $\left(\frac{r+1}{r}\right)|\mathcal{S}|mn + 2T - 1$ evaluations

Main Result

Achievability

Given parameters m, n, r such that $r|mn$, our scheme achieves

Recovery Threshold: $R = \left(\frac{r+1}{r}\right)|\mathcal{S}|mn + 2T - 1$

Normalized Download Cost: $\frac{R}{|\mathcal{S}|} \times \frac{\alpha^2}{mn} = \alpha^2\left(\frac{r+1}{r} + \frac{2T-1}{|\mathcal{S}|mn}\right)$

Normalized Computational Complexity: $\mathcal{O}\left(\frac{\alpha^3 r}{|\mathcal{S}|mn}\right)$

while guaranteeing batch size privacy against any T colluding workers

Main Result

Achievability

Given parameters m, n, r such that $r|mn$, our scheme achieves

Recovery Threshold: $R = \left(\frac{r+1}{r}\right)|\mathcal{S}|mn + 2T - 1$

Normalized Download Cost: $\frac{R}{|\mathcal{S}|} \times \frac{\alpha^2}{mn} = \alpha^2\left(\frac{r+1}{r} + \frac{2T-1}{|\mathcal{S}|mn}\right)$

Normalized Computational Complexity: $\mathcal{O}\left(\frac{\alpha^3 r}{|\mathcal{S}|mn}\right)$

while guaranteeing batch size privacy against any T colluding workers

Main Result

Achievability

Given parameters m, n, r such that $r|mn$, our scheme achieves

Recovery Threshold: $R = \left(\frac{r+1}{r}\right)|\mathcal{S}|mn + 2T - 1$

Normalized Download Cost: $\frac{R}{|\mathcal{S}|} \times \frac{\alpha^2}{mn} = \alpha^2\left(\frac{r+1}{r} + \frac{2T-1}{|\mathcal{S}|mn}\right)$

Normalized Computational Complexity: $\mathcal{O}\left(\frac{\alpha^3 r}{|\mathcal{S}|mn}\right)$

while guaranteeing batch size privacy against any T colluding workers

Main Result

Achievability

Given parameters m, n, r such that $r|mn$, our scheme achieves

Recovery Threshold: $R = \left(\frac{r+1}{r}\right)|\mathcal{S}|mn + 2T - 1$

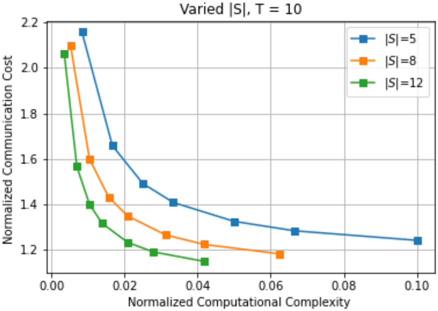
Normalized Download Cost: $\frac{R}{|\mathcal{S}|} \times \frac{\alpha^2}{mn} = \alpha^2 \left(\frac{r+1}{r} + \frac{2T-1}{|\mathcal{S}|mn}\right)$

Normalized Computational Complexity: $\mathcal{O}\left(\frac{\alpha^3 r}{|\mathcal{S}|mn}\right)$

while <u>guaranteeing batch size privacy against any T colluding workers</u>

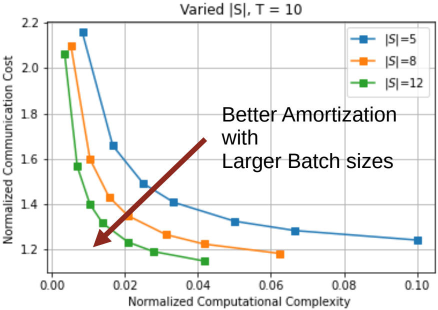
Experimental Simulations

- The following simulations fix the partitioning such that $mn = 24$.



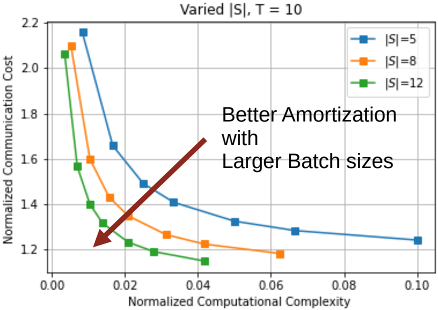
Experimental Simulations

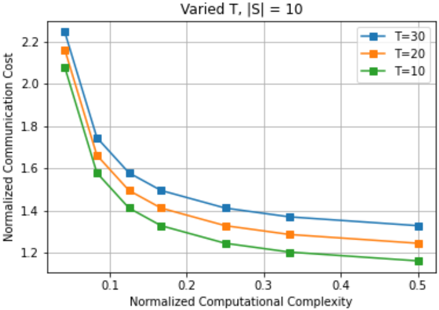
- The following simulations fix the partitioning such that $mn = 24$.



Experimental Simulations

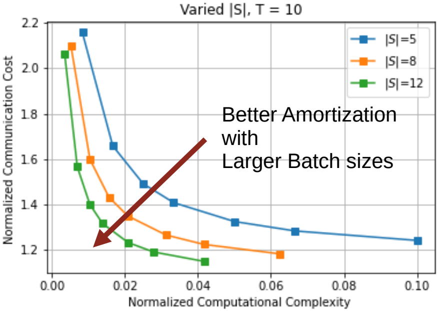
- The following simulations fix the partitioning such that $mn = 24$.

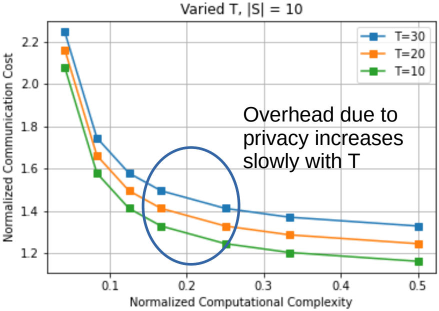




Experimental Simulations

➤ The following simulations fix the partitioning such that $mn = 24$.





Outline

Privacy from the Master (PFM)

- **Privacy from the master (PFM)** is another privacy constraint where the master cannot learn anything beyond what they requested

Privacy from the Master (PFM)

- **Privacy from the master (PFM)** is another privacy constraint where the master cannot learn anything beyond what they requested
- In the context of FPGMM, the master cannot learn anything more about the **matrix libraries** given the messages it receives, **beyond the information that $\mathbf{C}_{\mathcal{S}}$ provides**

Privacy from the Master (PFM)

- **Privacy from the master (PFM)** is another privacy constraint where the master cannot learn anything beyond what they requested
- In the context of FPGMM, the master cannot learn anything more about the **matrix libraries** given the messages it receives, **beyond the information that $\mathbf{C}_{\mathcal{S}}$ provides**
- Batch size privacy <u>incurs significant overhead</u> with current state of the art techniques for PFM

Current Approaches to Handling PFM

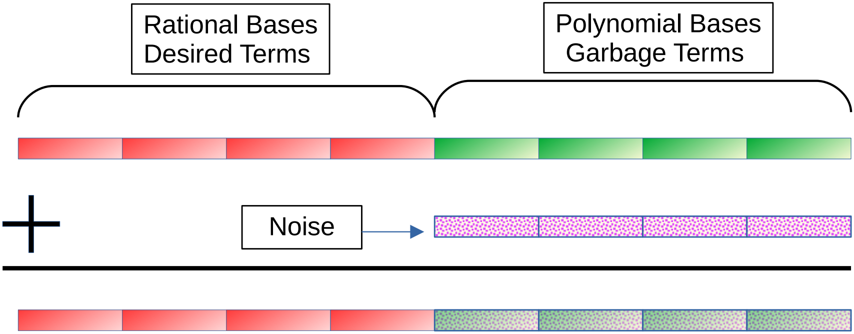
- Most coded computation techniques encode **desired terms** into certain **polynomial/rational basis functions**

Current Approaches to Handling PFM

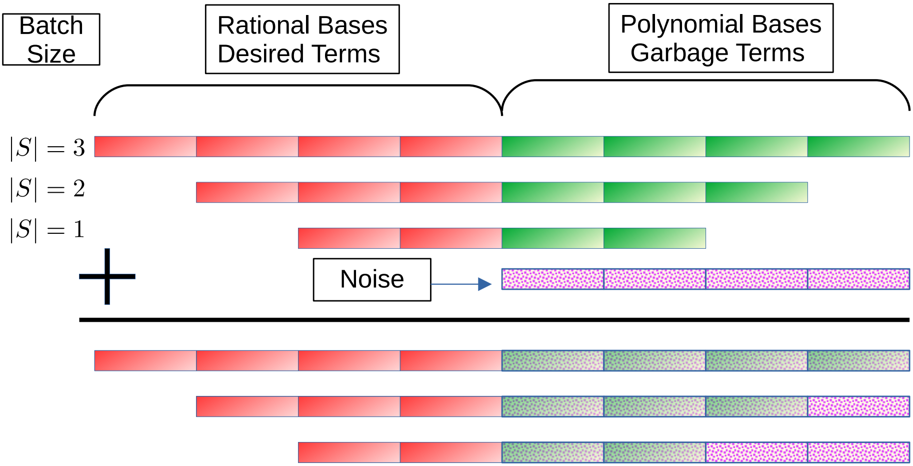
- Most coded computation techniques encode **desired terms** into certain **polynomial/rational basis functions**
- Thus, a natural solution to achieve PFM is adding **random noise** to the basis functions that **do not** contain the desired terms

Current Approaches to Handling PFM

- Most coded computation techniques encode **desired terms** into certain **polynomial/rational basis functions**
- Thus, a natural solution to achieve PFM is adding **random noise** to the basis functions that **do not** contain the desired terms
- Example: Generalized CSA codes with Noise Alignment (Chen '21) achieve PFM by adding noise to the polynomial terms (and a few rational terms)

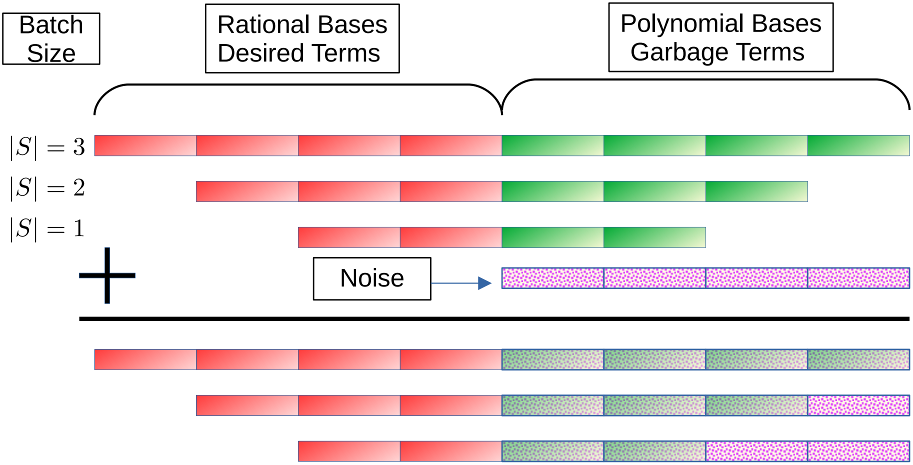


Privacy from the Master limits solutions to FPGMM



- To guarantee PFM, workers need to add a **pessimistic amount of noise** which significantly increases the recovery threshold

Privacy from the Master limits solutions to FPGMM



- We are currently researching novel techniques to address this issue based on recent advances in coded computation

Summary

- We initiated the first investigation into **batch size privacy** for coded computation

Summary

- We initiated the first investigation into **batch size privacy** for coded computation
- Introduced the **novel problem of FPGMM** that highlights the key issues of the new privacy model

Summary

- We initiated the first investigation into **batch size privacy** for coded computation
- Introduced the **novel problem of FPGMM** that highlights the key issues of the new privacy model
- We provided an achievable scheme utilizing CSA-like codes that **guarantees privacy, offers good straggler resilience, and provides flexible communication and computation costs**

Summary

- We initiated the first investigation into **batch size privacy** for coded computation
- Introduced the **novel problem of FPGMM** that highlights the key issues of the new privacy model
- We provided an achievable scheme utilizing CSA-like codes that **guarantees privacy, offers good straggler resilience, and provides flexible communication and computation costs**
- We highlighted that batch size privacy also complicates other privacy models such as **privacy from the master** and discuss our ongoing work into the topic

References

- (Gasca '89) M. Gasca et al.,"Computation of rational interpolants with prescribed poles", *Journal of Computation and Applied Math*, 1989
- (Olshevsky '01) V. Olshevsky and A. Shokrollahi, "A superfast algorithm for confluent rational tangential interpolation problem via matrix-vector multiplication for confluent cauchy-like matrices," *Structured Matrices in Mathematics, Computer Science, and Engineering I*, 2001.
- (Sun '18) H. Sun et al., "The capacity of private computation." *IEEE TIT* 2018
- (Raviv '19) N. Raviv et al., "Private polynomial computation from Lagrange encoding." *IEEE Transactions on Information Forensics and Security*, 2019
- (Kim '19) M. Kim et al., "Private coded matrix multiplication," *IEEE Transactions on Information Forensics and Security*, 2019
- (Yu '20) Q. Yu et al.,"Entangled Polynomial Codes for Secure, Private, and Batch Distributed Matrix Multiplication: Breaking the "Cubic" Barrier", *ISIT* 2020
- (Chen '21) Chen, Zhen, et al. "GCSA codes with noise alignment for secure coded multi-party batch matrix multiplication." *IEEE JSAIT*, 2021
- (Zhu '21) J. Zhu et al., "Improved constructions for secure multi-party batch matrix multiplication." *IEEE TCOM* 2021
- (Zhu '22) J. Zhu and S. Li, "A systematic approach towards efficient private matrix multiplication," *IEEE JSAIT*, 2022